**Lecture 3.**

**Sequences of real numbers. Limit of a sequences.**

**Monotonic sequences. Cauchy`s Convergence Criterion.**

**Sequences of Real Numbers**

An infinite sequence (more briefly, a sequence) of real numbers is a real-valued function defined on a set of integers We call the values of the function the terms of the sequence. We denote a sequence by listing its terms in order; thus,

(1)

The real number is the nth term of the sequence. Usually we are interested only in the terms of a sequence and the order in which they appear, but not in the particular value of k in (1). Therefore, we regard the sequences

as identical.

**Definition.** If for every positive number and there is such that , then the sequence  is called *convergent sequence* and a number ***а*** is called the *limit of sequence* and we write ***.***

A sequence that does not converge diverges, or is divergent.

**Uniqueness of the Limit**

**Theorem.** The limit of a convergent sequence is unique.

**Monotonic Sequence**

**Definition.** The sequence is said to be increasing if , nondecreasing if , decreasing if , nonincreasing if . A sequence that satisfies any of these conditions is called *monotonic*.

If the sequence  has only positive numbers, i.e.,   > 0 then monotonicity of sequence can be shown by conditions: the sequence is increasing if  > 1, the sequence is nondecreasing if   1, the sequence is decreasing if  < 1, and the sequence is nonincreasing if

  1.

**Theorem.** A convergent sequence is bounded.

**Definition.** A sequence is nondecreasing if for all n, or nonincreasing if for all n. A monotonic sequence that is either nonincreasing or nondecreasing. If for all n, then is increasing, while if for all n, is decreasing.

**Theorem.**

1. if is nondecreasing, then
2. if is nonincreasing, then

**Cauchy’s Convergence Criterion**

**Theorem** (Cauchy’s Convergence Criterion). The sequence  converges  if it is *fundamental*  * *>0  : ,   **  <**.